Damage Tolerant Design Using Collapse Techniques

Raphael T. Haftka*
Virginia Polytechnic Institute and State University, Blacksburg, Virginia

A new approach to the design of structures for improved global damage tolerance is presented. In its undamaged condition the structure is designed subject to strength, displacement, and buckling constraints. In the damaged condition the only constraint is that the structure will not collapse. The collapse load calculation is formulated as a maximization problem and solved by an interior extended penalty function. The design for minimum weight subject to constraints on the undamaged structure and for a specified level of the collapse load is a minimization problem which is also solved by a penalty function formulation. Thus the overall problem is a nested or multilevel optimization problem. Examples are presented to demonstrate the difference between the present and more traditional approaches.

Introduction

THE importance of designing structures to withstand damage has been recognized in the past, especially for combat aircraft. However, most of the work has been concentrated in predicting combat damage,[1] arresting crack propagation,[2] or analyzing the residual strength of the damaged structure.[3-5] Very little work has been done in the area of designing structures for tolerance to major damage, such as combat damage, except by cut-and-try methods.[6]

Recent work[7-10] has been directed toward developing the analytical tools for automated design subject to global damage tolerance (global damage is defined as the destruction of a major structural component as opposed to local damage such as a crack). The composite wing box examples used in Ref. 7 demonstrated the significant improvements in damage tolerance with no weight penalty that may be realized by including damage tolerance as a design constraint from the outset.

The work of Refs. 7-10 suffers from one major deficiency. The failure criteria used for the damaged structure are the same as those used for the undamaged structure, namely, the onset of yield or of buckling. While these failure criteria are reasonable for the undamaged configuration, they are not reasonable for the damaged structure. Once the structure suffers major damage, the concern of the designer is not about local yielding or buckling but about the ability of the damaged structure to carry the loads and not collapse. The exact prediction of the collapse load of a damaged structure calls for a costly nonlinear analysis. Such an analysis is difficult to incorporate in a design code because its repeated execution would be prohibitively expensive.

The procedure employed herein is to approximate the collapse load by employing a collapse technique such as the limit analysis method of the theory of plasticity. Such techniques assume that after yielding or buckling a structural member continues to carry a fixed load but has zero stiffness. This assumption reduces the computational cost because the equations of compatibility can be ignored. However, the use of a collapse technique transforms the problem of finding the collapse load of the damaged structure into a maximization problem. The design for minimum weight subject to constraints on the undamaged structure and a specified level of the collapse load is a minimization problem. Overall, the design problem becomes, therefore, a problem of nested or multilevel optimization. The present work employs the techniques of Ref. 11 for dealing with this multilevel optimization problem. The design procedure is demonstrated with truss and wing box design examples.

Received Oct. 22, 1982; revision received March 31, 1983.

*Professor, Aerospace and Ocean Engineering. Member AIAA.

Analysis and Design Procedure

The analysis procedure makes a distinction between a damaged and an undamaged configuration. For the undamaged structure the displacement formulation of the finite element method is used to write the equations of equilibrium as

$$Ku=L \qquad (1)$$

where K is the stiffness matrix, u the displacement vector, and L an applied load vector. The displacements are then used to calculate stresses and stress resultants. The constraints that are considered here are displacement constraints in the form

$$g_{di}(u)\geq 0 \qquad i=1,\ldots,n_d \qquad (2)$$

stress constraints

$$g_{si}(u)\geq 0 \qquad i=1,\ldots,n_s \qquad (3)$$

and buckling constraints

$$g_{bi}(u)\geq 0 \qquad i=1,\ldots,n_b \qquad (4)$$

For the damaged structure we assume that the equations of compatibility can be disregarded. That is, we assume that after yielding or buckling the internal loads begin to redistribute, with yielded and buckled elements becoming "soft" and undergoing large deformations. This situation is idealized here by assuming zero post-buckling or post-yielding stiffness. That is, the yielded or buckled element continues to carry the load that it carried at the onset of buckling or yielding, and that load does not increase with additional deformation. The collapse load is reached when no amount of internal load redistribution can balance the applied loads. These assumptions for obtaining the collapse load are conservative when the post-yielding behavior is ductile and the post-buckling behavior is stable. Otherwise, it can be nonconservative. Another assumption which is made here is that the changes in geometry are not severe; hence the equilibrium equations may be written in terms of the initial geometry. The above assumptions are implemented in the analysis by using element forces as the unknowns in the equations of equilibrium rather than the displacements. That is, the equations of equilibrium are written as

$$ET=fL \qquad (5)$$

where E is a matrix which depends only on the initial geometry of the structure, T a vector of element internal loads, and f a safety factor. For a statically indeterminate structure the matrix E is rectangular and it is not possible to determine T uniquely† from Eq. (5). Instead, based on the assumptions that we made, the structure will not collapse if it is possible to find a solution to Eq. (5) for $f=1$ such that the stress and buckling constraints are satisfied. These constraints are rewritten in terms of T as

$$g_{si}(T) \geq 0 \qquad i=1,...,n_s \tag{6}$$

$$g_{bi}(T) \geq 0 \qquad i=1,...,n_b \tag{7}$$

The constraint that the structure does not collapse at the applied load may be written as

$$f_{\max} \geq 1 \tag{8}$$

where $f_{\max}$ is the maximum value of f that may be obtained by selecting T to satisfy Eqs. (5-7).

This maximization problem is solved here by using a penalty function approach. An exterior penalty function is used for the equality constraints, Eq. (5), and an extended interior penalty function (Ref. 12) is used for the inequality constraints, Eqs. (6) and (7). Instead of maximizing f, we minimize $-fc$ where c is a constant. Using the penalty function formulation we minimize

$$P(T,f,r) = -fc + r\sum_{i=1}^{n_s} p(g_{si}) + r\sum_{i=1}^{n_b} p(g_{bi}) + \frac{c_1}{\sqrt{r}}\,||ET-fL||^2 \qquad \text{for} \quad r=r_1,r_2,...,r_i \to 0 \tag{9}$$

where the Euclidean norm is used and the extended interior penalty function is defined as

$$\begin{aligned} p(g) &= 1/g, && \text{if} \quad g \geq g_0 \\ &= 1/g_0[(g/g_0)^2 - 3(g/g_0) + 3], && \text{if} \quad g \leq g_0 \end{aligned} \tag{10}$$

where

$$g_0 = g_{01}(r/r_1)^{1/2} \tag{11}$$

The choice of the constants c, c_1 and the starting values of g_0 and r (i.e., g_{01} and r_1) are rather important for the conditioning of the problem. The first important consideration is that the contributions of the two types of penalty terms would be of comparable magnitude and that initially the fc term would also be of similar magnitude. This is achieved by selecting c to be the largest component of the applied load vector L, setting g_0 at 0.1, and setting

$$r_1 = c/30n_c \tag{12}$$

and

$$c_1 = c\sqrt{r_1}/n_T \tag{13}$$

where n_c is an estimated number of critical constraints ($g_{si}=0$, or $g_{bi}=0$, each having $p(g)=3/g_0$) and n_T the number of components in the vector T. The second important consideration is to have all variables in the minimization problem of comparable magnitude. This is accomplished by replacing f by fc as a minimization variable (as fc is of comparable magnitude to the components of T). The choice of r and $r^{1/2}$ as the multipliers of the interior and exterior penalty terms ensures that these terms remain of comparable magnitudes.[13] It also permits the use of extrapolation of the form

$$T = A + Br^{1/2} \tag{14}$$

which is the appropriate form for interior penalty function.[13] The vectors A and B are calculated on the basis of two values of r and are used to predict the minimizing values for the next value of r.

For each value of r the minimization of P is performed using Newton's method. For convenience we define $\bar{T}$ as the vector T augmented by the additional component fc. At the ith iteration Newton's method proceeds from $\bar{T}^i$ to $\bar{T}^{i+1}$ by solving

$$\frac{\partial^2 P}{\partial \bar{T}^2}\left(\bar{T}^{i+1} - \bar{T}^i\right) = -\left(\frac{\partial P}{\partial \bar{T}}\right)^T \tag{15}$$

where $\partial^2 P/\partial \bar{T}^2$ is the matrix of second derivatives $\partial^2 P/\partial \bar{T}_k \partial T_l$ (or Hessian) and $\partial P/\partial \bar{T}$ is the row vector of derivatives $\partial P/\partial \bar{T}_k$. The use of Newton's method is indicated for two reasons. First, the Hessian matrix $\partial^2 P/\partial \bar{T}^2$ is sparse and easy to calculate. The contribution of the equality constraints to it is constant and the contribution of each inequality constraint involves only a small number of components of T. Second, the Hessian matrix is needed for the calculation of the derivatives of the collapse load with respect to the design variables. Indeed, following Ref. 11 let us assume that P depends also on a design variable v, $P=P(T,r,v)$. The minimization of P gives us $T_m = \bar{T}_m(r,v)$ where the subscript m denotes T obtained by the minimization of P (i.e., $\bar{T}_m$ is the internal load vector and the value of fc corresponding to the collapse load). That is, T_m satisfies

$$\frac{\partial P}{\partial \bar{T}}(\bar{T}_m, v, r) = 0 \tag{16}$$

The derivative $\mathrm{d}\bar{T}_m/\mathrm{d}v$ is obtained by differentiating Eq. (16), which gives

$$\frac{\partial^2 P}{\partial \bar{T}^2}\frac{\mathrm{d}\bar{T}_m}{\mathrm{d}v} = -\frac{\partial^2 P}{\partial \bar{T}\partial v} \tag{17}$$

The overall problem may be formulated now as:

> Minimize $m(v)$ such that Eqs. (1-4) are satisfied for the undamaged structure, and Eq. (8) for the damaged structure

where m is the objective function taken here to be the mass of the structure. For this minimization problem we need derivatives of the constraints with respect to the design variables. Equation (17) provides us with derivatives of the collapse load. Derivatives of constraints on the behavior of the undamaged structure are obtained using standard (see Ref. 15) techniques.

The optimization method used for the overall problem employs the extended interior penalty function with an approximate form of Newton's method.[12] We thus have a nested or multilevel optimization method. One problem that is common in such a formulation (e.g., Ref. 14) is that the derivatives of an optimal parameter with respect to design variables may be discontinuous. This applies here to $\mathrm{d}\bar{T}_m/\mathrm{d}v$. Fortunately, with a penalty function approach this effect may be alleviated by using a fairly large value of r in Eq. (9). This keeps a large number of constraints active in the calculation

†For example, for the 25-bar truss shown in Fig. 1, there are 18 equilibrium equations (3 each for nodes 1 through 6) and 25 components of the vector T.

Table 1 Definition of 25-bar space truss

Material:	Aluminum	
Young's modulus:	E = 69 GPa	(10^7 psi)
Specific mass:	ρ = 2700 kg/m^3	(0.1 lbm/in.^{3})
Minimum area:	A = 0.065 cm^2	(0.01 in.^{2})

Allowable stresses

Members	Stress limits, MPa (psi) Compression	Tension	Members	Stress limits, MPa (psi) Compression	Tension
1	242 (−35,092)	276 (40,000)	12,13	242 (−35,092)	276 (40,000)
2-5	80 (−11,590)	276 (40,000)	14-17	46.6 (−6,759)	276 (40,000)
6-9	119 (−17,305)	276 (40,000)	18-21	46.6 (−6,959)	276 (40,000)
10,11	242 (−35,092)	276 (40,000)	22-25	76.4 (−11,082)	276 (40,000)

Nodal loading (2 load cases)

Load case	Node	Load components, kg (lbf) X	Y	Z
1	1	454 (1,000)	4,540 (10,000)	2,270 (−5,000)
	2	0	4,540 (10,000)	2,270 (−5,000)
	3	227 (500)	0	0
	6	227 (500)	0	0
2	5	0	9,070 (20,000)	2,270 (−5,000)
	6	0	9,070 (−20,000)	2,270 (−5,000)

Displacement limits for nodes 1 and 2 in X, Y, and Z directions = 0.89 cm (+0.35 in.)

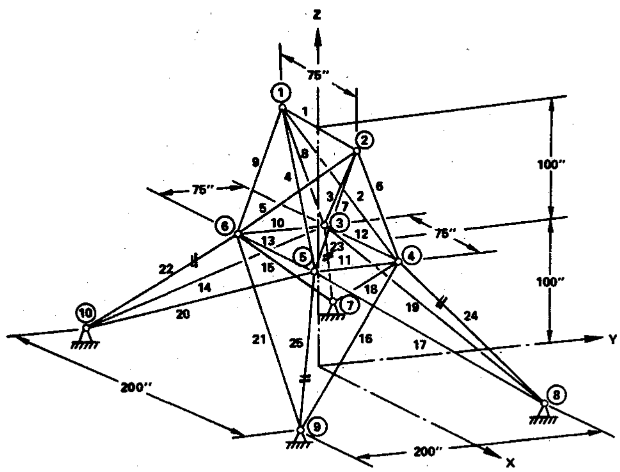

Fig. 1 Twenty-five-bar truss.

of the collapse load, and smoothes out the effect of critical constraints switching on the derivatives of the collapse load.

25-Bar Truss Design Example

The design procedure described in the previous section was applied to the 25-bar truss shown in Fig. 1. The material properties, loading, and allowables for the stress and displacement constraints are given in Table 1. Because of symmetry, only eight design variables are used. This is a very well known test problem for structural optimization algorithms, and the minimum mass design (without damage tolerance considerations) is known to be 247 kg (545 lb). In the present study the undamaged structure was designed to carry both loading conditions given in Table 1, while the damaged structure is required to carry only load condition 2.

The first damage condition that was considered was the destruction of members 22, 23, 24, and 25, as shown in Fig. 1. The truss was first designed employing the procedure of Ref. 7, which assumes that the damaged structure is designed subject to the same constraints as the undamaged structure (albeit under reduced loads). The minimum mass design for this case is 449 kg (939 lb), which is close to double the original undamaged minimum mass. By contrast, the present design procedure obtained a design of 280 kg (618 lb), which is only 12% heavier than the original design. An inspection of the two designs revealed that the large difference is due to the displacement constraints imposed on the conventional design rather than to the stress constraints. This means that the same design could be obtained by a conventional design procedure if the displacement constraint were not applied to the damaged structure.

A second damage condition that was analyzed was the destruction of member 7. In this case the displacement constraint was removed and a member buckling constraint based on Euler buckling was added. The moment of inertia used in calculating the Euler buckling load is assumed by the program to be of the form αA^2 where A is the cross-sectional area and α a proportionality constant (taken here to be 1). With these constraints the optimal design of the structure subject to no damage has a mass of 207.4 kg (457.3 lb). The conventional optimal design for damage tolerance requiring the structure to carry the second load condition when member 7 is broken has a mass of 212.7 kg (469.1 lb), which represents 5.3 kg (11.8 lb) mass penalty for improving the damage tolerance. The optimal design under a no-collapse constraint weighs only 207.6 kg (457.7 lb) which represents only 0.2 kg (0.4 lb) penalty for the improved damage tolerance. We thus see that the additional damage tolerance is obtained at a much lower mass penalty using the collapse approach.

Wing-Box Example

Description of Wing-Box Model

The unswept, untapered wing box with four spars and three ribs shown in Fig. 2 was used as the second example. The box is 3.56 m (140 in.) long, 2.24 m (88 in.) wide, and 38 cm (15 in.) deep. As shown in Fig. 2, the wing box is clamped at the root and a variable load is applied at the tip. The loads applied at the four tip nodes are 8163, 17,000, 17,000, and

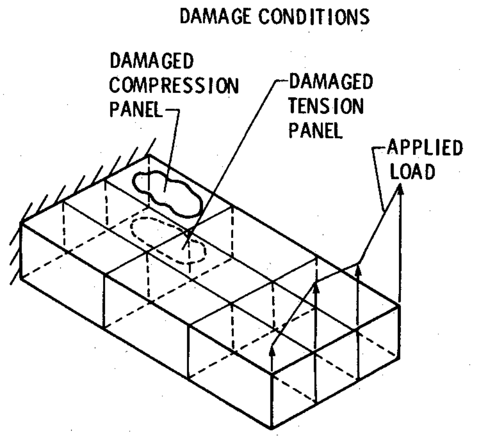

Fig. 2 Wing-box structure.

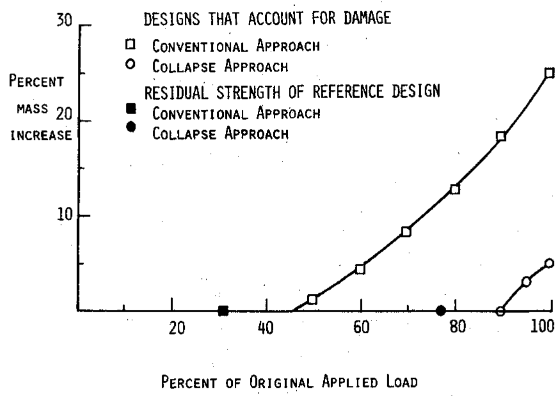

Fig. 3 Mass vs damage tolerance for tension panel damage.

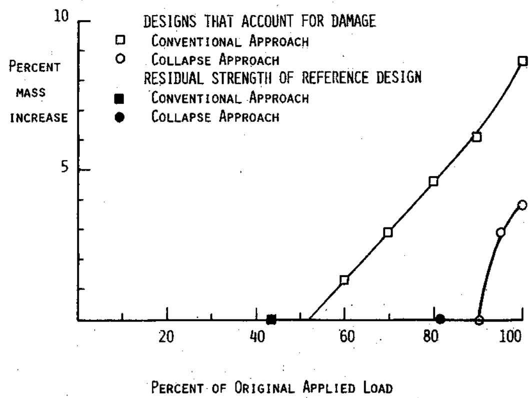

Fig. 4 Mass vs damage tolerance for compression panel damage.

34,000 kg (18,000, 37,500, 37,500, and 75,000 lb). The upper and lower wing-skin panels are modeled by membrane elements, and the webs of the ribs and spars are modeled by shear web elements. The spar caps and vertical posts at the rib-spar intersections are modeled by rod elements. The wing is assumed to be made of 7075 aluminum alloy.

The finite element model employs a single quadrilateral membrane element to represent a skin panel bounded by ribs and spars. The model has 32 grid points and 75 finite elements. Design variables are distributed so that there is one design variable for the thickness of each cover skin between adjacent ribs (i.e., constant chordwise distribution), one design variable assigned to the thickness of each spar web between ribs, one design variable assigned to the area of each spar cap between ribs, and one design variable assigned to the thickness of each rib. The cross-sectional areas of the vertical posts at the rib-spar intersections are held constant. The total number of design variables is 45. The design constraints are stress constraints [503 MPa (73,000 psi) allowable stress, using the Von Mises yield criterion], buckling constraints for panels and shear webs (see Ref. 15 for details of buckling analysis), and side constraints. The side constraints were 0.5 mm (0.02 in.) minimum gage for skin panels, 0.65 cm^2 (0.1 $in.^2$) minimum area for spar caps, and 3.9 cm^2 (0.6 $in.^2$) maximum area for spar caps.

Results and Discussion

The minimum-mass design of the wing was obtained first without considering any damage. The total structural mass of that design called herein the reference design was 669 kg (1475 lb). Then two damage conditions were considered. The first is the destruction of one of the upper or compression panels near the root (see Fig. 2). The second damage condition is the destruction of the corresponding lower or tension panel of the wing box.

The wing-box example was employed in Refs. 7 and 8 to demonstrate the advantage of including damage tolerance constraints in the design process. The minimum mass design of the wing, obtained without any consideration of damage, (i.e., the reference design) was shown in Ref. 7 to be able to carry only about 30% of the load when the tension panel is destroyed and about 44% of the load when the compression panel is destroyed. These percentages are based on the conventional damage design approach whereby the damaged structure is subject to the same constraints as the undamaged one. Reference 7 also reports on a series of damage-tolerant designs that require the structure to carry 100% of the loads in the undamaged condition and a specified percentage of these loads when damaged. These damage-tolerant designs included a design that could carry about 46% of the load when the tension panel is destroyed and a design that could carry about 53% of the load when the compression panel is destroyed; both designs having the same mass as the reference design! The increase in damage tolerance without increase in mass is explained by the fact that in many design situations it is possible to find many optima of similar weights with different combinations of design variables. When damage-tolerance is not included as a constraint the actual design that is obtained depends on the details of the optimization procedure. When such constraints are included the optimization procedure picks the most damage-tolerant design.

The same procedure was repeated herein employing the no-collapse constraints for the damaged structure. The results for the tension-panel damage are shown in Fig. 3. The unfilled symbols in Fig. 3 show the mass increase over the reference design required for a structure to carry 100% of the loads when undamaged and a specified percentage of the loads when damaged. The filled symbols show the residual strength of the reference design. It is seen from Fig. 3 that the reference design can carry about 77% of the original loads before collapse, compared to 30% of the loads before initial yielding or buckling. When the structure is designed with the collapse load constraint it can carry about 90% of the load before collapse without any mass increase. The mass increase for a design that can carry 100% of the loads with tension-panel damage is about 5%, compared to about 25% when the conventional damage approach is used.

Figure 4 shows the corresponding results for designs that are tolerant to compression panel damage. The results are similar to those of the tension-panel damage except that this case is a less severe damage case so that the mass penalty of damage-tolerant designs is lower.

Concluding Remarks

A design procedure for obtaining the optimum design of structures subject to global damage tolerance constraints has been presented. The procedure is based on a collapse technique that estimates the collapse load of the damage structure. The mathematical formulation leads to a nested optimization problem which requires the computation of derivatives of an optimum with respect to the problem parameters.

The procedure has been applied to the design of a 25-bar truss and a wing-box structure. It is shown that the proposed procedure results in a lower mass penalty for achieving damage tolerance than the more conventional approach. It has also been demonstrated that (as in the case of conventional design for damage tolerance) inclusion of the proposed method in the design cycle may improve the damage tolerance without additional mass.

Acknowledgment

This work was supported by NASA Grant NAG-1-168.

References

[1] Burch, G. T. and Avery, J. G., "An Aircraft Structural Combat Damage Model," AFFDL-TR-70-116, Nov. 1970.

[2] McCarty, J. E. and Johnson, R. W., "An Approach to the Development of Durable and Damage Tolerant Composite Commercial Aircraft Structure," AIAA Paper 77-471, 1977.

[3] Steely, R. W. and Melosh, R. J., "Structural Analysis, Frailty Evaluation and Redesign (SAFER)," AFFDL-TR-70-15, Aug. 1970.

[4] Venkayya, V. B., Khot, N. S., and Eastep, F. E., "Vulnerability Analysis of Optimized Structures," *AIAA Journal*, Vol. 15, Nov. 1978, pp. 1189-1195.

[5] Switzky, H., "Rapid Analysis of Damaged Structures," *AIAA Journal*, Vol. 14, 1976, pp. 1159-1161.

[6] Jensen, J., Thorndyke, P., and Campbell, M., "Aircraft Wing Structural Concepts with Improved Ballistic Damage Tolerance," Convair Division of General Dynamics, Rept. GDC-DDG69-002, Oct. 1969.

[7] Starnes, J. H. Jr. and Haftka, R. T., "Preliminary Design of Composite Wing-Box Structures for Global Damage Tolerance," AIAA Paper 80-0755, 1980.

[8] Haftka, R. T., Starnes, J. H. Jr., and Nair, S., "Design for Global Damage Tolerance and Associated Mass Penalties," *Journal of Aircraft*, Vol. 20, Jan. 1983, pp. 83-88.

[9] Sun, P. F., Arora, J. S., and Haug, E. J., "Fail Safe Optimal Design of Structures," *Journal of Engineering Optimization*, Vol. 2, No. 1, 1976, pp. 45-53.

[10] Arora, J. S. and Govil, A. K., "Optimal Design of Large Structures for Damage Tolerance," *AIAA Journal*, Vol. 18, May 1980, pp. 563-570.

[11] Sobieszczanski-Sobieski, J., Barthelemy, J. F., and Riley, K. M., "Sensitivity of Optimum Solutions of Problem Parameters," *AIAA Journal*, Vol. 20, Sept. 1982, pp. 1291-1299.

[12] Haftka, R. T. and Starnes, J. H. Jr., "Applications of a Quadratic Extended Penalty Function for Structural Optimization," *AIAA Journal*, Vol. 14, June 1976, pp. 718-724.

[13] Fiacco, A. V. and McCormick, G. P., *Nonlinear Programming Sequential Unconstrained Minimization Techniques*, John Wiley & Sons, New York, 1968.

[14] Barthelemy, J. F. and Sobieski, J., "Sensitivity Analysis of the Optimum Design of Composite Panels," *AIAA Journal*, Vol. 21, May 1983, pp. 797-799.

[15] Starnes, J. H. Jr. and Haftka, R. T., "Preliminary Design of Composite Wings for Buckling Strength, and Displacement Constraints," *Journal of Aircraft*, Vol. 16, Aug. 1979, pp. 564-570.